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On Neutrino Oscillations and Flavour Persistence

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Abstract

Using the quantum-mechanical amplitude analysis of an unstable system in motion, we investigate the persistence of flavors for the neutrino propagation and the emergence of the relativistic effect.

Key words: neutrino, quantum mechanics and special relativity.

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The notion of particle masses, especially for neutrino, remains an actual problem of particle physics where neutrinos have been a field of wide studies [1, 2, 3]. In the SM, neutrinos were presented as verily massless fermions, in contrast with the recent evidences of their flavour oscillations bringing the first sign of the non-zero tiny neutrino masses [4, 5, 6]. This makes the propagation behaviour of neutrinos an important toul in investigating various subjects beyond the standard model of particle physics [7, 8, 9].
In relativistic motion, it is well known that the lifetime of an unstable system is increased with respect to the same system at rest by the relativistic factor[1] [10, 11, 12, 13]. This standard relativistic increase of the lifetime of an unstable system in motion follows from elementary considerations based on the relativistic time dilation and on the physical requirement that the fact that a system has evolved or not does not depend on its velocity with respect to the observer. In case of neutrino propagation, the evolution of the flavour states is subject to the relativistic effect and hence the persistence of flavours.

In what follows we start with a brief review of the quantum mechanical treatment of an unstable flavour neutrino in the rest frame, then we discuss its extension to the relativistic motion and finally we investigate the emergence of the relativistic effect.

In non-relativistic quantum mechanics, the persisted amplitude that an unstable neutrino flavour $f = e, \mu, \tau$ in motion with a relativistic velocity $\overrightarrow{v}$ described by the state $\left| v^f_{\overrightarrow{v}} \right\rangle$ is not evolved at the time t is [10],

$$A^f_{\overrightarrow{v}}(t) = \left\langle v^f_{\overrightarrow{v}} \right| e^{-iHt} \left| v^f_{\overrightarrow{v}} \right\rangle , \tag{1}$$

where H is the Hamiltonian operator. Since it is always possible to expand the state $\left| v^f_{\overrightarrow{v}} \right\rangle$ over the eigenstates of the Hamiltonian, we write,

$$\left| v^f_{\overrightarrow{v}} \right\rangle = \sum_f C_{p_f, m_f} \left| p_f, m_f \right\rangle , \tag{2}$$

with C_{p_f, m_f} are complex coefficients. the state $\left| v^f_{\overrightarrow{v}} \right\rangle$ is defined by,

$$\begin{aligned} H \left| p_f, m_f \right\rangle &= E_f \left| p_f, m_f \right\rangle \\ &= \sqrt{p_f^2 + m_f^2} \left| p_f, m_f \right\rangle , \end{aligned} \tag{3}$$

with the normalization,

[1] With v is the velocity of the relativistic particle and the velocity of light taken $c = 1$, the corresponding relativistic factor is $\gamma = (1 - v^2)^{1/2}$.

$$\langle p_f, m_f \left| p'_{f'}, m'_{f'} \right\rangle = \delta_{mm', p_f p_{f'}}. \tag{4}$$

In the rest frame $\overrightarrow{p}_f = \overrightarrow{0}$, the unstable flavour neutrino is described by,

$$\left| v^f_{\overrightarrow{0}} \right\rangle = \sum_f C_{m_f} \left| m_f \right\rangle, \tag{5}$$

Here $\left| C_{m_f} \right|^2 = \left| C_{p_f=0, m_f} \right|^2 = |\langle m_f \left| v^f_{\overrightarrow{0}} \right\rangle |^2$ is the distribution of mass (energy in the rest frame) of the flavour neutrino, which determines the persisted amplitude of the flavour f 1,

$$A_f^{\overrightarrow{0}}(t) = |\langle m_f \left| v^f_{\overrightarrow{0}} \right\rangle |^2 e^{-i m_f t}, \tag{6}$$

In a reference frame in which the neutrino is in motion with velocity $\overrightarrow{v}$, we have $\overrightarrow{p}_f = \gamma m \overrightarrow{v}$ and the neutrino state is transformed by the unitary $U_{\overrightarrow{v}}$ operator,

$$U_{\overrightarrow{v}} \left| v^f_{\overrightarrow{0}} \right\rangle = \left| v^f_{\overrightarrow{v}} \right\rangle, \tag{7}$$

such that,

$$U_{\overrightarrow{v}} \left| p_f = 0, m_f \right\rangle = \left| p_f = \gamma m v, m_f \right\rangle. \tag{8}$$

Therefore, in this reference frame, the system is now described by the state 2 as,

$$\left| v^f_{\overrightarrow{v}} \right\rangle = \sum_f \langle p_f, m_f \left| v^f_{\overrightarrow{v}} \right\rangle \left| p_f, m_f \right\rangle. \tag{9}$$

now, to obtain the correct persisted amplitude of the falvour f, we must take into account both the time and space evolutions of the system within the evolution operator; the relation 1 becomes then,

$$A^f_{\overrightarrow{v}}(t) = \left\langle v^f_{\overrightarrow{v}} \right| e^{-i(Ht - Px)} \left| v^f_{\overrightarrow{v}} \right\rangle, \tag{10}$$

where P is the momentum operator whose the action is,

$$P \left| p_f, m_f \right\rangle = p_f \left| p_f, m_f \right\rangle, \tag{11}$$

at this level, if we accept that the neutrino coordinate reads $\overrightarrow{x} = \overrightarrow{v} t$ since it is moving with velocity $\overrightarrow{v}$, we finally obtain from the relation 10 the desired persisted amplitude of the flavour f[2],

[2]Note that using $x = \overrightarrow{v} t$ we obtain the desired quantity as a function of time only as in the case of the rest frame.

$$A_f^{\overrightarrow{v}}(t) = |\langle m_f \left| v_{\overrightarrow{0}}^f \right\rangle|^2 e^{-im_f t/\gamma}, \tag{12}$$

involving the relativistic factor γ. Indeed, confronting with the survival amplitude 6 of the system at rest, we end up with,

$$A_f^{\overrightarrow{v}}(t) = A_f^{\overrightarrow{0}}(t/\gamma). \tag{13}$$

where one can see that the relativistic effect influences the neutrino flavour changing amplitude during its propagation.

In this work, we have shown that the correct mesure of the persisted amplitude of an oscillating flavour during neutrino propagation is subject to relativistic effect. In particular, we have shown that elementary considerations based on relativistic time dilation lead to the standard relativistic relation 13 between the flavour survival amplitudes of neutrino in motion and at rest. This relativistic effect has been obtained with a relativistic approach taking into account, in addition to the time evolution as done usually, the space evolution of the unstable neutrino flavours.

The phenomenon of neutrino oscillations still deserve deep investigations. The connection with other interpretations such as extra-dimensions, quantum gravity,... could bring more insights in the future.

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