Newtonian-Riccati-Dirac Dual Solution Spaces

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In 1723 J. Riccati studied a non-linear differential equation of the form $y' + ay^2 = bx^\alpha$. It was solved by D. Bernoulli (1724) using elementary functions for $\alpha = -2$ or $\alpha = -4k(k-1)$ where k is an integer. L. Euler (1769) showed that a generalized Riccati equation is equivalent to a linear homogeneous differential equation of second order. J. Louiville (1841) proved that for certain values of α the solution cannot be expressed in quadratures of elementary functions. In this paper we will (1) look at how Newton's laws of motion can be expressed in the form of a Riccati Equation, (2) examine the Riccati form of the Dirac equation in relation to the fractional electric charges of quarks, and (3) address the question whether quantum mechanics can be "reformulated in terms of nonlinear Riccati equations." The solution spaces of the Newtonian, Riccati, and Dirac equations are dual. Knowing a special solution of the Riccati equation we can write the general solution. From the general solution we can find a second special solution which generates another general solution analytically and geometrically distinct from the first one. Investigation of the second general solution reveals that it resides in an 'antispace' which contains information as valid as the original general solution space but is inaccessible from it. The two solution spaces effectively cancel each other out and satisfy a zero-totality condition.