Iterants, Fermions and the Dirac Equation I and II

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Abstract. The simplest discrete system corresponds directly to the square root of minus one. The square root of minus one is a clock, when it is seen as an oscillation between plus and minus one. This is the way of thinking about the square root of minus one as an iterant. More generally, by starting with a discrete time series of positions, one has immediately non-commutativity of observations since the measurement of velocity involves the tick of the clock and the measurment of position does not demand the tick of the clock. Commutators that arise from discrete observation generate a non-commutative calculus, and this calculus leads to a generalization of standard advanced calculus in terms of a non-commutative world. In a non-commutative world, all derivatives are represented by commutators. In this view, distinction and process arising from distinction is at the base of the world. Distinctions are elemental bits of awareness. The world is composed not of things but processes and observations. We will discuss how basic Clifford algebra comes from very elementary processes. Clifford algebra is at the base of the world. We show how the simple algebra of the split quaternions, the very first iterant algebra that appears in relation to the square root of minus one, is in back of the structure of the operator algebra of the electron. The underlying Clifford structure describes a pair of Majorana Fermions, particles that are their own antiparticles. These Majorana Fermions can be symbolized by Clifford algebra generators a and b such that $a^2 = b^2 = 1$ and $ab = - ba$.
One can take a as the iterant corresponding to a period two oscillation, and b as the time shifting operator. Then their product ab is a square root of minus one in a non-commutative context. These are the Majorana Fermions that underlie an electron. The electron can be symbolized by Phi = $a + ib$ and the anti-electron by Phi* = $a - ib$. These form the operator algebra for an electron. Note that PhiPhi* = $(a + ib)(a + ib) = a^2 + b^2 + i(ab + ba) = 0 + i0 = 0$.
This nilpotent structure of the electron arises from its underlying Clifford structure in the form of a pair of Majorana Fermions. We discuss how this nilpotent structure in related to the Dirac equation and draw a direct connection with the work of Peter Rowlands. We then discuss how braiding is related to the Majorana Femions and to the Fibonacci model for universal topological quantum computation. In this model, Majorana fermions appear as collective vortices of electrons that have non-trivial braiding relationhips. It is hoped that such phenomena will be comfirmed in the Fractional Quantum Hall Effect.

These two talks will be self-contained, with the more complicated quantum topology occurring in the second talk.