Knot Theory and Physical Theory

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Abstract. The purpose of this talk is to explore a number of relationships between knot theory and physical theory. These range from the physical properties of knots (adding structure to the topological model such as thickness and bendability of rope - hence differential geometry, and then frictional properties) to relationships with statistical mechanics, quantum theory and quantum gravity.
These more theoretical relationships arise from the structure of knot invariants such as the Jones polynomial. We develop them first by graph theory and combinatorics of link diagrams. Then we introduce matrices and solutions of the Yang-Baxter equation and relationships with statistical mechanics. All of this is developed in an elementary way and we will be self-contained with both the mathematics and the associated physics. Then we reformulate these invariants again using the formalism of functional integrals and gauge fields. This allows us to discuss Witten's relationships of quantum field theory with link invariants. It also allows us to explain and detail the amazing reformulation of general relativity due to Ashtekar that reframes it so that there is gauge theory in back of the metric. With the Ashtekar reformulation in hand we can then show how Witten's functional integral approach to the Jones polynomial leads to solutions to a version of so-called loop quantum gravity due to Ashtekar, Smolin and Rovelli. Ultimately, it is the purpose of this talk to formulate this relationship of knot theory with quantum gravity and to explore its properties and meanings.