Abstract

The search is for a natural correspondence between music and category theory. Music is a composition of sounds from point to point as a succession of transitions. Category theory also involves, as a central tenet, the principle of composition, from the target object of one arrow to the source object of another. In both music and category theory the arrows have a direction from a starting point to a closing point, though loops may exist.

The practice by a performer of playing a score is the personal communication, often highly intensive, of the piece to a listener. When performing a player is at the same time both looking forward to what is to be played next and looking back at what has just been played. The process of music is indeed similar to that for transactions in a database system, where monads have been used to represent process [1]. There are however some significant differences. Aestheticism is an important part of music, covering aspects of articulation such as style and improvisation, subject to the rules of intonation. So while in database systems it would be a major deficiency if transactions were not always perfect to the letter of the requirements, in music variation through expression is an integral part of a performance, involving a departure from the score in aspects such as phrasing, rubato and articulation.

The intension/extension relationship plays a central role in music. The philosophical basis comes from Aristotle and Frege [2]. The intension is the type; the extension is the collection of instances that satisfy the type. It is not as simple though as a hierarchy of types. There remains a philosophical dimension to the design. The Universe contains everything. The Universe of Discourse (UoD) is that section of the Universe of interest to our application. By the laws of physics we cannot isolate any part of the Universe but we can identify a section for our work. In this case the intension is the Universe and the extension, UoD, is the world of music. A musical manuscript is extensional to the UoD of Music as one of the objects in this universe. The intended physical sounds are intensional to the sounds delivered in an actual performance, which forms the ultimate extension in our structure.

The genre of the music has a bearing on the typing and hence on the nature of the intension/extension relationship. We consider popular music, film music, chants and jazz. Music is often viewed as discrete, particularly when played on the piano but can be continuous with string instruments in microtones and in digital instruments. Our approach presented here is suitable for any discrete music, based on semitones, and can provide the flexibility to handle scores where expressiveness is sought. We bring together these ideas in the formal definition of the topos for a score as data structure and of the monad as process, operating inside the topos. Handling continuous music is work in progress.
